

RELATIONSHIP BETWEEN DEGREE OF BILINGUALISM AND MATHEMATICAL WORD PROBLEM SOLVING

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ABSTRACT

The purpose of this study was to determine the extent to which degree of bilingualism influences bilingual students' reversal error generation on compare word problems that have two different levels of consistency in two languages of testing. It was hypothesized that bilingualism has positive, cognitive consequences depending on the levels of linguistic proficiency reached in both languages. Also, the consistency hypothesis assumed that inconsistent language problems in which the unknown variable is the object of the relational sentence are more error-prone than consistent language problems in which the unknown variable is the subject of the relational sentence. Seventy seventh graders and 55 eighth graders from two urban school districts were subjects in the study. Two mathematics tests were used as outcome measures to assess subjects' mathematical problem solving performance in both Haitian and English. The results of this study showed that both bilingualism and language consistency significantly influenced student performance in word mathematical problem solving.

Introduction

Bilingualism, defined as possessing two languages, has always been a controversial issue in society. During the early 1900s, bilingualism was considered an unwelcome topic

among American professionals and politicians. Educators rendered bilingualism responsible for immigrant children's failure in school subject matter. Employers believed that immigrants, due to their low competence in English, did not fit the requirements needed to become part of the United States workforce. Psychologists regarded bilingualism as a handicap to cognitive development; it was assumed that bilingualism was a barrier affecting verbal intelligence (Darcy, 1963). Policy makers who accorded great importance to psychologists' works declared that bilingual education should be banned in schools and at the workplace since it did not benefit learning growth and productivity in industry and business. Foes of bilingualism and bilingual education elicited support for their assumption; they wanted to demonstrate that bilingualism was a handicap to the functioning of society and turned to social science research (e.g., Saer, 1923). The result of such a research was accepted with little attention given to how the results were attained.

Early bilingual researchers did not consider the psychometric standards that are presently guiding the conduct of research (Diaz, 1985; Nanez, Padilla, & Lopez-Maez, 1992). For example, researchers used tests that were developed for the White American middle-class. Therefore, scores that were obtained from these tests had to be interpreted according to norms relevant to the White American population. The use of these tests with the bilingual population would immediately minimize their validity and reliability. The majority of bilinguals, living in the United States at that time, were recently arrived from Mexico, Asia, and non-English-speaking European countries; these immigrants were from different cultures and social classes. The assessment of cognitive abilities of immigrants with translated tests was viewed as violating basic principles of measurement and research. Peal and Lambert (1962), who conducted a study

in which they controlled for age and socio-economic factors, found evidence that bilingualism had positive effects on cognition, contrary to the results of Saer's (1923) study. Peal and Lambert (1962) found that the "balanced bilingual" enjoyed a "mental flexibility, a superiority in concept formation, and a more diversified set of mental abilities" (p. 3). Peal and Lambert, however, failed to demonstrate in their study the cognitive consequences of different levels of bilingualism.

Other scholars were concerned with the methodological flaw in Peal and Lambert (1962) study. Diaz (1985) suggests that studies on bilinguals should be conducted within class membership in order to freely observe the effects of levels of bilingualism on cognition. At present, the majority of bilingual studies follow Diaz's (1985) suggestions for studying the effects of bilingualism in specific cognitive areas including mathematics. Several studies (Clarkson & Galbraith, 1992; Hernandez, 1983; Tuck, 1983) have found support for the positive relationship between bilingualism and mathematics; however, the literature on bilingualism includes few studies examining the effect of degree of bilingualism in mathematical word problem solving and the ways in which balanced bilinguals demonstrate their "mental flexibilities" when faced with challenging mathematical word problems expressed in two languages of testing (as referred to languages in which a test is administered). Research by Lewis and Mayer (1987) and Stern (1993) showed that compare word problems, one of the four classes of arithmetic word problems, are very difficult for children (Riley & Greeno, 1985). Compare word problems are characterized by a static relation between two variables. They come in two special classes: consistent language problems in which the unknown variable is the subject of the second sentence, and inconsistent language problems in which the unknown

variable is the object of the second sentence. The solution of inconsistent language problems requires the rearrangement of the relational sentence to fit the format of the relational sentence of consistent language problems. The straightforward prediction is that reversal error will be likely to occur during mental transformation of inconsistent language problems. There are no studies that look at levels of bilinguals and compare word problem solving. This study is the first to investigate the influence of degree of bilingualism in compare word problem solving.

Language and Bilingualism

Language plays a major role in thinking. It serves as a mediator for the connection of thoughts and ideas. Vygotsky (1962) asserts that inner speech provides an individual with the opportunity to examine the liaison that takes shape between identical elements that several separate thoughts share together. The higher-order thought that results from the merge of these several individual thoughts allows a thinker to explore new areas of ideas (Anderson, 1995). The role of language is more than a passive host for the enhancement of thinking. It plays an active role in the production of metacognitive thinking. It creates a state of alertness for the thinker to check his or her productive thoughts. Thus, a lack of language proficiency can limit a thinker's awareness of contemplating new ideas that can emanate from his previous thoughts or monitor his thinking process (Baker, 1995; Bialystok, 1988). Cummins (1987) also entertains the cognitive advantages of bi-lingual proficiency. He asserts that bilingual individuals who reach a minimum level of language proficiency in both first and second languages are capable to demonstrate great thinking skills. Greater range of language proficiency would depend on the extent to which a bilingual person can transfer his thoughts from one language to the other. Coffeen (1982) asserts a strong command in two languages would lead an individual to more

linguistic information, greater storage and retrieval of information abilities, and the abilities to contrast linguistic systems in developing conceptual thought processes. However, Cummins (1987) recognizes the cognitive limitations of bilingualism when a bilingual person does not develop yet the minimal level of language proficiency in either his first language or the second language. In the same vein, Cummins hypothesizes that individuals who develop high linguistic proficiency in both first and second language (called balanced bilinguals) are capable to enjoy the positive cognitive benefits of bilingualism. Needless to say, bilingualism can have a positive influence on problem solving since it is considered as the highest form of cognition (Mayer, 1991). Several studies explored the relationship between bilingualism and arithmetic problem solving (Mestre, 1986)

Mathematical word problem solving

Mathematical word problems consist of story problems that incorporate numbers. These story problems always end with question(s). The success of a problem solver is related to his ability to accurately represent the situation that is being told in the story and to select the most appropriate mathematical operation to find the solution (Kintsch & Greeno, 1985). The representation of a word problem is broken into two parts (Mayer, 1991). First, the problem solver decodes the textual components of the problem text into propositional representations. Next, he integrates these representations with his existing knowledge domain to create a broader representation of the problem. During this process, the problem solver actively searches his previous knowledge to select a model that can best integrate the linguistic composition of the problem text and the contextual situation that emanates from the text. Thus, an accurate

translation of a problem text is as important as the problem solver's previous knowledge in understanding the problem.

The degree of difficulty met during the comprehension process varies with the type of word problems. The literature counts four problem prototypes (Carpenter & Moser, 1982; Riley & Greeno, 1988; Riley et al., 1983). They respond to change problems in which a set of objects is transformed to another set, combine problems in which two sets of objects merge together to become a larger set, and compare problems in which two sets of objects are compared.

Interestingly, compare word problems are to determine one of the two sets of objects, given that one set of objects and the difference set of objects are known.

In particular, compare word problems are built with a relational statement that brings together the two sets to be compared (Lewis & Mayer, 1987). It is this relational statement that is the locus of attention in representing compare word problems. The relational statement can be presented in two symmetric forms that would require the same mathematical operation that leads to the solution of the problem. In one form, the relational statement is consistent with the problem solver's information processing, in which the unknown variable is the subject of the statement (e.g., *John has five cookies. Janet has two cookies less than John. How many cookies does Janet have?*). Thus, problems of this form are called consistent language problems. In the other form, the relational statement is inconsistent with the problem solver's information processing, in which the unknown variable is the object of the relational statement (e.g., *John has 5 cookies. He has two cookies more than Janet. How many cookies does Janet have?*). Word problems that pertain to this form are called inconsistent language problems.

Lewis and Mayer (1987) hypothesize that problem solvers are more likely to use a mathematical operation that is irrelevant to the situation described in the problem when solving inconsistent language problems than when solving consistent language problems. They coin a problem solver's *mis*-representation of an inconsistent language problem as reversal error. A problem solver will commit reversal error when she failed to mentally rotate the relational statement of the inconsistent language problem to fit the relational statement of the equivalent consistent language problem. Thus, the problem solver tends to use a mathematical operation that is relevant to the situation described in the problem text.

Test Language

In measurement and research, test language refers to the language in which subjects are tested. A test taker is expected to respond to test items in the same manner whether they are written in the first language or second language. Psychometric qualities of a test can be negatively affected if the test language does not convey clearly the objectives of the test items. The negative consequences of test language are more felt when a test is designed to assess students' achievement or intelligence in a language in which they do not have a strong command (Gunnarsson, 1978). Such an inconvenience would increase measurement errors that will, in turn, minimize the reliability and validity of the test. Gunnarson (1978) states " Psychometric tests are clearly biased against speakers of non-majority varieties of English" (p. 34). To control the effect of test language on test takers' achievement, tests must be administered and assessed in takers' first and second languages (Pletcher et al., 1978; Simon et al., 1977).

Purpose of the Study

The purpose of this study was to determine the extent to which degree of bilingualism influences bilingual students' reversal error generation on compare word problems having two different levels of consistency in two languages of testing. More specifically, the study is designed to answer these research questions:

1. Does bilingualism influence reversal error generation on mathematical compare word problems?
2. Does test language affect production of reversal errors on mathematical compare word problems?
3. Does language consistency influence generation of reversal errors on mathematical compare word problems?
4. Is the effect of bilingualism on reversal error commitment moderated by test language?
5. Is the effect of bilingualism on reversal error commitment moderated by language consistency?
6. Is the effect of language consistency on reversal error generation moderated by test language?
7. Is the effect of bilingualism on reversal error commission moderated by the interaction of test language and language consistency?

METHOD OF THE STUDY

Participants

The study was conducted using 125 subjects chosen from two urban school districts. These subjects were Haitian/English bilingual students living in the urban school district. They were 70 seventh graders and 55 eighth graders attending their last month of the school year. Four-fifths of the students who participated in the study were born in Haiti. The length of residence of the subjects in the United States of America varied from 1 to 12 years. An examination of the data from the personal data questionnaire revealed that English-dominant students have been living in the United States for 3 to 9 years, balanced bilingual students for 1 to 5 years, and Haitian-dominant students for 1 year to 4 years. The average age of the subjects was 13 years.

Since the sample was made up of two age groups, it is assumed that age or mental development would be an extraneous factor that could affect the relationship between bilingualism and compare word problem solving. Therefore, the investigator examined the association between grade (seventh and eighth grade) and error category. The results of the investigation showed that developmental growth was not a major threat to the results of the study since seventh and eighth graders tended to achieve differentially in only two out of eight compare problem types.

Preparation of Writing Topics and Questionnaire

The investigator prepared two topics from which the subjects were to choose one on which to write in Haitian in one period and in English in another period. Since the content of a

topic was irrelevant, the investigator selected the optional topics that could offer the subjects the greatest latitude in expressing their ideas through writing skills (abundant vocabulary, grammatical rules, organization and sequences of ideas, illustration of ideas in appropriate examples). In the topic, the students were asked to make use of the relational terms (more than, less than, as many) that were relevant to the content of the word problems they were going to solve. The contents of the topics “shopping” and “household work” were based on the fact that most of the students had experienced shopping and witnessed various activities their relatives did at home. Students did not have to search for novel ideas that would delay their writing ability.

The other screening measure was a questionnaire that the investigator prepared for students to rate their own proficiency in both Haitian and English. The four linguistic skills (listening, speaking, writing, and reading) in both languages were the areas of self-evaluation. A four-option scale was used, ranging from “Very Well,” “Well,” “Quite Well,” to “Not At All.

Administration of Screening Measures

The writing activity embraced two important dimensions: timing and language distribution. Students wrote on a topic in Haitian in one period and in English in another period. The regular 60-minute period was given for each time. Also, the investigator divided the subjects into two groups of equal numbers. One group was requested to write their topic in Haitian, the other group in English. At the end of the first period, the investigator collected all students’ papers. During the second period, the groups switched writing language. The group who had been writing in Haitian now wrote in English; the group who had been writing in English now wrote in Haitian. If someone needed additional time to complete a composition, the investigator accorded the student a 5- to 10-minute extension of time.

In the prelude to the essay writing, the investigator distributed to the students the sheets containing the topic on which to write (the topic was written in both Haitian and English) and two blank sheets of paper. Instructions about the topic were read to the subjects. The classroom teacher assisted the investigator in encouraging the whole class to collaborate and participate in the writing activity which was an important part of the research study.

The second section of the screening was the language questionnaire which took place during another time. The questionnaire completion was done in 25 minutes. First, the investigator distributed the questionnaires to the students to fill out. He then instructed them how they would proceed in completing the questionnaire. Next, the investigator read the questions in both Haitian and English, and the subjects checked the answer box that was appropriate to their feelings. The subjects were required to listen carefully to the instructor before they checked any answer box. When the students answered all items in the questionnaire, the investigator collected the filled-out questionnaires while the students remained in their seats.

Scoring of Screening Measures

To score students' writing tests, the investigator used the following criterion writing skills: abundant vocabulary, grammatical rules, organization and sequence of ideas, and illustration of ideas in appropriate examples. Students who showed high performance on those skills in both languages (Haitian and English) were tentatively classified as balanced bilinguals. Students who showed high performance on the criterion skills in only one language were tentatively categorized as Haitian-dominant or English-dominant bilinguals, depending on the language of dominance. Students who showed poor performance in both languages were considered limited bilinguals.

To score the questionnaires, the investigator categorized students' responses ("Very Well" and "Quite Well") as "favored" and ("With Many Errors" and "Not At All") as "not-favored." Students who favored both Haitian and English for speaking, listening, writing, and reading were tentatively classified as balanced bilinguals. Students who favored Haitian more than English for speaking, listening, writing, and reading were tentatively classified as Haitian-dominant. Finally, students who favored English more than Haitian for speaking, listening, writing, and reading were tentatively classified as English-dominant.

The three categories of bilingual students resulting from the direct assessment (writing proficiency test) were compared with the three categories of bilingual students obtained from the indirect observation (questionnaire) to evaluate the consistency of the two instruments. It resulted that 92 students were classified as either balanced bilingual, English-dominant, or Haitian-dominant students by both assessing instruments. The investigator used the results of the writing test alone to categorize the other nine students whose placements did not match by both the writing test and the self-report questionnaire. Three of these students were categorized as Haitian-dominant, and the other six students were classified as English-dominant. A total of 36 students were classified as balanced bilingual, 32 students as Haitian-dominant, and 33 students as English-dominant. Finally, the remaining 24 students who demonstrated poor performance in both languages on the writing tests were not considered for further categorization. This group was rejected from the study.

Problem Solving Tests

The investigator prepared the test items based on Lewis and Mayer's (1987) Consistency Hypothesis. Each problem comprised two steps: a compare problem as the first step and a direct variation problem as the second step. The compare portions of the problems correspond, respectively, to eight problem types: consistent addition (i.e., requires adding two numbers and the problem says "more"), inconsistent addition (i.e., requires adding two numbers and the problem says "less"), consistent subtraction (i.e., requires subtracting the second number from the first and the problem says "less"), inconsistent subtraction (i.e., requires subtracting the second number from the first and the problem says "more"), consistent multiplication (i.e., requires multiplication of two numbers and the problem says "n times as many"), inconsistent multiplication (i.e., requires multiplication of two numbers and the problem says "1/n times as many"), consistent division (i.e., requires division of two numbers and the problem says "1/n times as many"), and inconsistent division (i.e., requires division of two numbers and the problem says "n times as many").

Each problem consisted of three sentences. The first sentence was an assignment statement expressing the value of some variable (e.g., "At Apple Tree butter sells 65 cents per stick"). The second sentence was a relational statement, expressing the value of a second variable in relation to the first variable (e.g., "Butter at C-Town sells 8 cents more per stick than butter at Apple Tree"). The third sentence asked a question about the value of some quantity in terms of the second sentence (e.g., "If you want to buy 15 sticks of butter, how much will you pay at C-Town?"). Answering the question always involved multiplication of the value of the second variable by a quantity given in the third sentence (Verschaffel et al., 1992).

The materials (either words or numbers) that comprised the tests were chosen by the investigator to be relevant to the subjects' previous knowledge, since the purpose of the study was not to evaluate students' vocabulary knowledge and arithmetic operations. In consultation with an expert in applied linguistics, the investigator examined the cultural and grammatical relevance of the words that entered in the construction of each problem so that they would not affect the subjects' understanding process. The arithmetic operations that were used in the tests (addition, subtraction, multiplication, and division) were familiar to the population of seventh and eighth graders so that they were not a major obstacle to the subjects' comprehension of the test items.

Test Administration

The investigator distributed the mathematics tests in two groups. Each group had eight mathematics problems (four English and four Haitian) in which the language of testing was alternated (English, Haitian, English, Haitian, etc.). In the first group, the mathematics test items were of the following types: consistent addition, inconsistent addition, consistent subtraction, and inconsistent subtraction. In the second group, the mathematics test items were of the following types: consistent multiplication, inconsistent multiplication, consistent division, and inconsistent division. In each group, the items were arranged in four different blocks so that each item would have the same chance of being presented to the examinees. The four blocks (made of ordered items) were:

- (a) 1, 2, 3, 4, 5, 6, 7, 8;
- (b) 8, 7, 6, 5, 4, 3, 2, 1;
- (c) 4, 3, 2, 1, 8, 7, 6, 5;

(d) 5, 6, 7, 8, 1, 2, 3, 4.

Each block made a test booklet that contained nine regular sheets of paper, with one arithmetic word problem typed on each sheet and a front sheet that enlisted the test instructions to the examinee. Also, each block of items within each separate group was equally distributed to the examinees.

The administration of the mathematics tests was done at two different times. During the first time, the examinees were presented with consistent addition, inconsistent addition, consistent subtraction, and inconsistent subtraction problems in both Haitian and English. During the second time, examinees were given consistent multiplication, inconsistent multiplication, consistent division, and inconsistent division problems in both Haitian and English. The order of the testing times was different for each participating school. The investigator read the testing instruction to the examinees and informed them that they had a 60-minute period to finish the test; nevertheless, students who needed more time to complete the test were given extra time. The classroom teacher assisted the researcher in encouraging the students to try each problem, and especially to show all their work.

Scoring of Problem Solving Tests

Each mathematics exam was scored “correct” if the proper numerical answer was given; otherwise, the solution was scored as belonging to one or more of the following error categories: “reversal error” is a representation error in which the numerical operation for the first step in the problem is reversed (i.e., addition for subtraction, subtraction for addition, multiplication for division, and division for multiplication); “arithmetic error” occurs when the computation in the first or second step of the problem is carried out incorrectly, and “goal

monitoring error” occurs when the second step (i.e., direct variation) of the problem is omitted. The investigator computed frequencies and percentages of each category of error types and examined the occurrence of reversal errors in order to respond to the research questions.

Further Scoring of Problem Solving Tests

Four scores were computed for each subject based on the two languages of the test (English and English) of the consistent and inconsistent problem types. In other words, subjects were given an English consistent score (Y1), Haitian consistent score (Y2), English inconsistent score (Y3), and Haitian inconsistent score (Y4). Since there were four consistent and inconsistent problems presented in each of the two languages, a subject could be scored 0, 1, 2, 3, or 4 if he/she solved none of the four problem types incorrect, one of them incorrect, two of them incorrect, three of them incorrect, or all four incorrect, respectively, within each consistency level and each language level.

Design of the Study

The design of the study was a repeated measures design consisting of one between-subjects factor and two within-subjects factors. The between-subjects factor was bilingualism with three levels (Haitian dominance, balanced bilingualism, and English dominance). The within-subjects factors were language consistency with two levels (consistent language and inconsistent language) and test language with two levels (English and Haitian). The dependent variable of the study was reversal errors. Hence, the design was a $3 \times 2 \times 2$ repeated measures of analysis of variance.

RESULTS AND DISCUSSION

The present study sought to examine the relationship between reversal error generation in mathematical compare word problem solving and degree of bilingualism, language consistency, and test language. The results of the study were consistent with Cummins' (1976) Threshold Theory and Lewis and Mayer's (1987) Consistency Hypothesis. The data showed that dominant bilingual students (mean = 2.20), more than balanced bilingual students (mean = 1.13), generated a greater proportion of reversal errors in solving compare word problems, $[F(1, 98) = 52.03, p < .05]$. Similarly, the data showed a significant main effect between English-dominant and Haitian-dominant students on compare word problems, $[F(1, 98) = 8.28, p < .05]$. The results indicated that English-dominant students (mean = 1.96) were less likely than Haitian-dominant students (mean = 2.46) to commit reversal errors on compare word problems.

The error generation on compare word problems demonstrated that the sample of balanced bilingual students and that of the dominant bilingual students used in the study represented two different subpopulations (having different cognitive and linguistic abilities) of the bilingual population. This finding was in line with previous studies (Cathcart, 1980, 1982; Clarkson & Galbraith, 1992; Dawe, 1983; Hernandez, 1983; Tuck, 1983) seeking to show evidence to the threshold hypothesis (to demonstrate the cognitive superiority of balanced bilinguals, as compared to non-balanced bilinguals, in mathematics). This finding gave supported to Arnberg's (1981) argument that cognitive development is additive, and that the further the child moves towards balanced bilingualism, the more he accumulates cognitive advantages. Although one would intuitively believe that balanced bilinguals are cognitively more flexible than dominant bilinguals, the finding may be surprising given the fact that the

United States Policy has in general supported programs whose goal is transitional rather than full bilingualism (Hakuta, 1987).

The data revealed that bilingual students tended to commit more reversal errors when solving inconsistent language problems (mean = 2.71) than when solving consistent language problems (mean = .98), $[F(1, 98) = 244.27, p < .05]$. In other words, bilingual students were more prone to commit reversal errors (e.g., using an arithmetic operation that was primed by the relational term in the problem) when solving inconsistent language problems than when solving consistent language problems that already fit their preferred format (Lewis & Mayer, 1987). This finding corroborated with previous studies in compare word problem solving, showing that students committed more reversal errors on inconsistent language problems than on consistent language problems (Cummins et al., 1988; Hegarty et al., 1992; Lewis & Mayer, 1987; Morales et al., 1985; Riley & Greeno, 1988; Riley et al., 1983; Stern, 1993).

The examination of the significant two-way interaction between language consistency and bilingualism illuminates the impact of the independent variables (bilingualism and language consistency) on reversal error generation. In the study, the trend of reversal error generation across the three levels of bilingualism was not linear across the two levels of language consistency. The study showed that the direction of the error difference between balanced and dominant bilingual students remained the same across both consistent and inconsistent language problems. In other words, balanced bilingual students committed less reversal errors than dominant bilingual students on both consistent and inconsistent language problems, $F(1, 98) = 25.60, p < .05$; however, the data showed that the direction of error difference between Haitian-dominant and English-dominant students changed across consistent and inconsistent language

problems. In other words, Haitian-dominant students tended to generate a higher rate of reversal errors on consistent language problems (mean = 1.89) than English-dominant students (mean = .68), while the latter tended to commit a higher rate of reversal errors on inconsistent problems (mean = 3.03) than did Haitian-dominant students (mean = 3.21). This finding further provided evidence that balanced bilinguals' superior problem solving capability over dominant bilinguals was not influenced by the degree of difficulty of the mathematical problems at hand. However, the superior performance of Haitian-dominant students over English-dominant students was influenced by the degree of difficulty of compare word problems. While English-dominant students showed greater problem-solving ability on consistent language problems, as compared to Haitian-dominant students, the latter were more proficient than English-dominant students on inconsistent problems that were more challenging than consistent problems. The unequal performance of the two dominant bilingual groups on compare word problems came encounter to the assumption that students whose linguistic proficiency is below the second threshold level in both languages should have demonstrated the same cognitive strengths when faced with challenging academic endeavors (Cummins, 1987). There are three various plausible explanations for the uneven performance between Haitian- and English-dominant bilingual students in inconsistent language problem solving. First, the interactive effect of bilingual dominance with language consistency in reversal errors could be an artifact of the sample in the study. The subjects came into the study as preselected Haitian-dominant and English-dominant, with their own previous backgrounds that were not controlled. Prior schooling, unequal tutoring service given to the students at home, and students' socio-economic status and individual mathematical ability could account for the unequal performance. The second explanation for the

uneven compare word problem performance between Haitian- and English-dominant students could be due to the nature of arithmetical thinking that characterized the problems that comprised the tests. The problems were additive and multiplicative types. It was suggested that multiplicative thinking that is more cognitively demanding than additive thinking could have had an interactive effect with the relationship between bilingual dominance and language consistency. The third explanation for the superiority of Haitian-dominant students over English-dominant students in inconsistent language problems could be provided through the analysis of linguistic components (personal pronoun, key-words, language symmetry) of compare word problems that is addressed later in this paper.

This study found that test language did not influence bilingual students' reversal error generation. Students committed reversal errors on compare problem solving across both English and Haitian versions of the test with the same magnitude. This finding was not consistent with the assumption that the majority language more than the minority language would favor a greater acceptance of learning and experience (Coffeen, 1982; Cummins, 1976). One would expect the subjects in this study to be more prone to generate more reversal errors in Haitian (the minority language) than in English (the majority language). This finding sheds light on the possible syntactic similarities between English and Haitian that capture the comprehension of compare word problems. For example, subjects might represent at an equal rate, in both English and Haitian, the phrases "more than," "less than," and so forth in their model of problem comprehension. Further research studies are suggested to take on the relationship between the syntactic and semantic structures of English and Haitian in the context of mathematical word

problems in order to be more accurate about the influence of test language on word problem solving.

The interpretation of the two-way interaction between test language and bilingualism better explains the main effects of the independent variables on the dependent variable (reversal error generation). The data showed no significant interaction between bilingualism and test language, ($p > .05$). In other terms, the direction of error difference between balanced and dominant bilingual students and between Haitian-dominant and English-dominant students remained the same across both levels (English and Haitian) of test language. Even though the interaction was not significant, it was worth studying since it would provide a better way to interpret the main effects already discussed. Indeed, balanced bilingual students tended to generate fewer reversal errors than dominant bilingual students, whether the language of the test was English or Haitian. This finding showed that balanced bilingual students' superior performance in compare problem solving, as compared to dominant students, was not influenced by either English and Haitian across both consistent and inconsistent language problems.

The data revealed no interaction between language consistency and test language, ($p > .05$). The direction of the main effect vector (consistency vs. inconsistency) of the language consistency factor was the same across both levels (English and Haitian) of test language. In other words, subjects tended to commit more reversal errors when solving inconsistent language problems than when solving consistent language problems across both language versions (English and Haitian) of the mathematics tests. Again, Lewis and Mayer's (1987) consistency hypothesis was supported by the inherent characteristics of inconsistent language problems to cause more reversal errors than consistent language problems in compare word problem solving,

regardless of the language version of the test. In other words, the possibility to err in rearranging relational terms in inconsistent problems has more to do with logico-mathematical knowledge that requires problem solvers to mentally manipulate the symmetry of these operations (n more x than y and n less y than x) than with linguistic knowledge that simply rests upon the meanings of more, less, and than separately. Indeed, logico-mathematical knowledge invariably persists through natural language while linguistic knowledge may vary from one language (e.g., English) to another (e.g., French).

CONCLUSIONS

The results of the study showed that degree of bilingualism had a significant, positive relation with error generation on compare word problems. The data revealed that both Haitian- and English-dominant students were more likely than balanced bilingual students to generate reversal errors in compare word problems. This finding gave support to the assumption that balanced bilingualism is causal to cognitive development (Arnberg, 1981). However, while Haitian-dominant students were more likely than English-dominant to commit reversal errors on consistent language problems, the latter outperformed English-dominant students in inconsistent language problem solving that are assumed to be more difficult than consistent language problems. Three explanations were given to the fact that Haitian-dominant students generated less reversal errors than English-dominant students on inconsistent language problems. First, the two dominant bilingual groups were preselected in the study; they came with their own prior knowledge and mathematical ability that were not controlled. It is suggested that these uncontrolled factors might contribute to the unequal performance of the Haitian- and English-dominant groups. Second, the nature of the arithmetical thinking on the problems tests could

also influence the uneven performance of the two groups. Since the mathematics tests were comprised of additive and multiplicative problems, it is suggested that multiplicative thinking could have an interactive effect with the relationship between bilingual dominance and language consistency. Third, the complex processing of linguistic components (personal pronoun, key-words, and language symmetry) of compare word problems may also explain the unequal performance between Haitian- and English-dominant students. It seems that Haitian-dominant students, as compared to English-dominant students, processed at a greater rate the search for a referent to the personal pronoun that is embedded in inconsistent language problems, the representation of inconsistent language problems that is model-centered instead of key-word strategies, and the rearrangement of the relational sentence of inconsistent language problems to fit the format of the second sentence of consistent language problems that is parallel to human cognitive processing.

The analysis of variance showed that test language did not have a significant relation with reversal error generation on compare word problems. In other words, students tended to commit reversal errors in compare word problems at an equal rate in both Haitian and English versions of the test. This result was not in line with the assumption that the majority language would be more likely than the minority language to favor a greater acceptance of learning and experience (Coffeen, 1982; Cummins, 1976). One would expect the subjects in the study to be more prone to commit reversal errors in Haitian (the minority language) than in English (the majority language). This finding suggested linguistic similarities between English and Haitian that embrace the comprehension of compare word problems. The interaction between bilingualism and test language was not significant. In other words, the reversal error differences

found among the three bilingual groups did not depend upon the level of test language, whether English or Haitian. Again, this finding showed that the cognitive superiority of balanced bilinguals was not influenced by test language; the cognitive advantages of balanced bilinguals allowed them to transcend the superficial, linguistic embodiment of word problems to attain the situation that emanates from the text input (Kintsch & Greeno, 1983). The data revealed no significant interaction between language consistency and test language factors. Subjects tended to generate more reversal errors when solving inconsistent language problems than when solving consistent language problems, whether the test language was Haitian or English. This finding lent support to the inherent nature of inconsistent language problems to be more error prone than consistent language problems, regardless of language of testing (Lewis & Mayer, 1987). Finally, the data showed no significant three-way interaction among bilingualism, language consistency, and test language. The two main effects of bilingualism that captured the error differences between balanced and dominant bilingual students and between Haitian-dominant and English-dominant students did not statistically vary with the interaction between language consistency and test language.

Changes to the Study

Methodological changes that should be effected in the study are relevant to test item composition, data collection, test scoring, and sample analysis. An increase of test items is important for test reliability improvement. The regulations of schools in which the study was conducted allowed the investigator to use a short period of time in order to intervene as little as possible with students' learning time. The investigator was forced to administer a short test that could fit a 60-minute period. Thus, each of the eight problem types could appear only once on

the test. Test takers could succeed or fail each problem type only once. The scoring grading showed that the test reliability could be hampered as students had reduced opportunity to show their genuine understanding of each problem type. Chance might play a great role in students' decision making in answering the questions. Hence, it is highly suggested to re-conduct the same study in an environment where a lengthened test, that will be more reliable, can be used in the research study (Anastasi, 1986). Improved reliability would enhance the validity of the test, which would make more precise the relations that are sought between bilingualism, test language, language consistency, and reversal error generation.

The second way of improving the relation between bilingualism and compare problem solving would be to minimize foreign factors that would affect data collection and measurement. There are a number of determinants of error variance in this study such as noise, students' end-of-year preoccupations (final exams, graduation rehearsal, and field trips), teachers' indifference or excessive zeal in motivating their students to participate in the study, age, grade level, and a single examiner to correct students' exams. The effects of these factors are twofold: a reduction of experimental variance and a decrease in the reliability of the writing and mathematics tests. "Thus minimizing error variance has two principal aspects: (1) the reduction of errors of measurement through controlled conditions of an experiment, and (2) an increase of the reliability of measures" (Kerlinger, 1986, p. 290). Conducting the same study at the beginning of the school year or during the mid-school year is encouraged since these are times when students' attention is preoccupied with fewer school-related duties (e.g., final exams) in a noise-free atmosphere. The subjects would be drawn from the same age or grade population. Also, it is suggested that teachers be informed to keep a neutral and unobtrusive behavior during test

administration. While teachers' interest in the study (in terms of encouraging students to participate in the study) is appreciated, they may influence students' scores in providing them cues (verbal or body gestures).

Likewise, one examiner can influence students' scores for different reasons: fatigue, misreading of a sentence, and so forth. In replicating the same study, it is suggested that two examiners grade students' papers; they would then compare the two sets of scores to see if they measure the same thing (e.g., reversal error) with the same scale. The given recommendations would reduce error variance and give systematic variance (e.g., experimental variance due to bilingualism) a chance to show itself.

In the present study, it is assumed that the preselection of bilingual groups could account for their unequal performance in compare word problem solving. In replicating the study, it is suggested that one control the sample more effectively; one would collect data on subjects' prior schooling (whether bilingual or not), date of arrival in the United States, tutoring service given after school or at home, socio-economic status, and mathematical ability.

Educational Implication of the Study

The results of the study can be used in two ways: fostering bilingual education in the school system and training dominant bilingual students to represent mathematical compare word problems. The outstanding performance of balanced bilingual students over dominant bilingual students in compare problem solving indicated the positive cognitive benefits of bilingualism in academic endeavors. Therefore, it is suggested that United States educational policy makers should encourage bilingual education in the educational system. Funds should be made available to bilingual programs in order to provide bilingual students with appropriate learning

environments and competent bilingual educators who will allow them to develop their cognitive abilities in academic endeavors.

The second way of implementing the results of the study (whereby balanced bilingual students outperformed dominant bilingual students in compare problem solving) is to train dominant bilingual students to represent mathematical word problems. Previous studies have indicated that the majority of students' errors in word problems are misrepresentations of problem structure rather than computational errors (Anand & Ross, 1987; DeCorte, Verschaffel, & DeWinn, 1985). It is suggested that teaching dominant bilingual students in their two languages how to represent word problems may improve their performance in compare word problems.

Further Research

With respect to future research in mathematics and bilingual education, the results of the present study suggest the following questions:

1. To what extent do linguistic components (the personal pronoun, key words, and language symmetry) of compare word problem influence the relation between degree of bilingualism and reversal errors?
2. To what extent do problem-types (additive and multiplicative) influence the relation between degree of bilingualism and compare word problem solving?
3. To what extent do first-language dominant and second-language dominant bilingual students demonstrate the same cognitive strengths in solving compare word problems?
4. To what extent do linguistic similarities between two languages (e.g., Haitian and English) influence mathematics problem solving learning among bilinguals?

5. Does the relationship between degree of bilingualism and mathematical word problem solving persist across all age and grade levels?

These questions await other investigators.

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